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On Some Properties of a Generalized Geometric Distribution

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ABSTRACT
This study introduces a two-parameter generalization of the geometric distribution and investigates its fundamental statistical properties. Closed-form expressions for the probability generating function, cumulative distribution function, mean, variance, mode, index of dispersion, survival function, and hazard rate function are derived. The problem of parameter estimation is addressed using the method of maximum likelihood, and a generalized likelihood ratio test is developed to examine the significance of the additional parameter. The practical usefulness of the proposed distribution is illustrated through applications to real medical data sets. Furthermore, a Monte Carlo simulation study is carried out to assess the finite-sample performance of the estimators. The results demonstrate that the proposed model provides greater flexibility and improved data fitting compared to some existing count data models.

KEYWORDS
count data modeling; probability generating function; maximum likelihood estimation; geometric distribution; survival function; simulation

1. Introduction

The global outbreak of COVID-19 has generated an unprecedented amount of data related to daily infections, recoveries, and fatalities. Accurate statistical modeling of such data plays a crucial role in understanding disease dynamics and in supporting effective public-health decision making. Since many COVID-19 related observations are recorded as discrete counts, continuous probability models are often inappropriate for analysis. Consequently, discrete distributions have received considerable attention for modeling such real-life count data.

Several authors have proposed new discrete models and their extensions to capture different dispersion patterns observed in practice. However, it has been observed that many existing models fail to provide satisfactory fits for certain medical and epidemiological data sets, particularly when the data exhibit complex dispersion behavior. This limitation motivates the development of more flexible discrete distributions.

In this paper, we propose a new two-parameter generalization of the geometric distribution, referred to as the generalized geometric distribution (GGD). The additional

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parameter enhances the flexibility of the classical geometric model and allows it to accommodate a wider range of dispersion structures.

The remainder of the paper is organized as follows. Section 2 presents the definition of the proposed distribution along with several of its mathematical properties. Parameter estimation using the method of maximum likelihood and hypothesis testing procedures are discussed in Section 3. Section 4 illustrates the practical applicability of the model using real medical data sets, while Section 5 provides a brief simulation study to evaluate the performance of the estimators. Finally, Section 6 concludes the paper with a summary of major findings.

2. Definition and Properties of GGD

A non-negative integer valued random variable X is said to follow the generalized geometric distribution (GGD) if its probability mass function (p.m.f) is of the following form, for any $q \in (0,1)$, $\theta > 0$ and x= 0,1,2,... .

$$g(x) = \Delta_0 \, \frac{q^x \; x! \; \Gamma(\theta)}{\Gamma(\theta + x)}, \tag{1}$$

where

$$\Delta_0 = [{}_2F_1(1,1;\theta;q)]^{-1},$$

and

$${}_2F_1(a,b;c;u) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n u^n}{(c)_n n!}.$$

Here ${}_2F_1(a,b;c;u)$ is the Gauss hypergeometric function. For further details regarding Gauss hypergeometric function, refer Mathai and Haubold (2008). Here $(a)_n$ denote the Pochhammer's symbol: $(a)_0$=1, $(a)_n$= $a(a+1)...(a+n-1)$, for n= 1,2,3,... and $(a)_n$= $\frac{\Gamma(a+n)}{\Gamma(a)}$, for $a > 0$, where $\Gamma(.)$ denotes the gamma function.

Clearly when $\theta = 1$, the GGD reduces to the standard geometric distribution with parameter q.

Proposition 2.1. *The probability generating function (p.g.f) of the GGD is given by*

$$H(t) = \Delta_0 \; {}_2F_1(1,1;\theta;qt). \tag{2}$$

Proof. By definition, the p.g.f of the GGD with p.m.f g(x) is given by

$$H(t) = \sum_{x=0}^{\infty} g(x) t^x \tag{3}$$

$$= \Delta_0 \sum_{x=0}^{\infty} \frac{x! \; q^x \Gamma(\theta)}{\Gamma(\theta + x)} t^x, \tag{4}$$

which gives (2). □

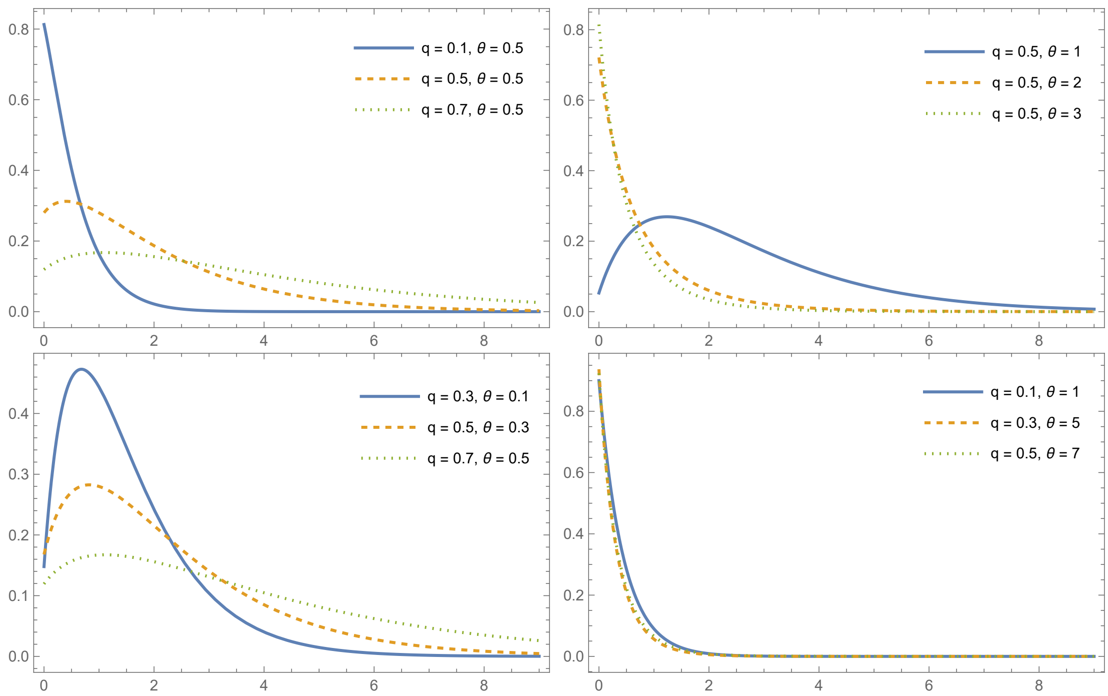

Figure 1. Illustration of the p.m.f of GGD for different values of θ and q.

Proposition 2.2. *The cumulative distribution function (c.d.f) G(x) of the GGD has the following form, for any* $x \in \Re$.

$$G(x) = 1 - \triangle_0 \; q^{x+1}\frac{(2)_x}{\theta(\theta+1)_x} \; {}_2F_1(1, 2+x; \theta+x+1; q) \tag{5}$$

Proof. By definition, the c.d.f of the GGD with p.m.f (1) is

$$\begin{aligned} G(x) &= P(X \leq x) \\ &= \triangle_0 \sum_{k=0}^{x} \frac{(1)_k}{(\theta)_k} q^k \\ &= 1 - \triangle_0 \sum_{k=x+1}^{\infty} \frac{(1)_k}{(\theta)_k} q^k \\ &= 1 - \triangle_0 \; q^{x+1}\frac{(1)_{x+1}}{(\theta)_{x+1}} \sum_{k=0}^{\infty} \frac{(2+x)_k}{(\theta+x+1)_k} \; q^k, \end{aligned}$$

which leads to equation (5). □

Proposition 2.3. *An expression for factorial moments of the GGD is given by*

$$\mu_{[n]} = \frac{q^x \; x! \; (1)_x}{(\theta)_x}\delta_j \tag{6}$$

where $\delta_j = \Delta_0 \; {}_2F_1(j+1, 1+j; \theta+j; q)$ *for* $j = 1, 2, 3, \ldots$.

Proof. The factorial moment generating function of the GGD with p.g.f (2) is

$$F(t) = H(1+t) = \Delta_0 \left[{}_2F_1(1,1;\theta;q(t+1))\right] \tag{7}$$

On differentiating (7) n times with respect to t and putting t=1, we get (6). □

Proposition 2.4. *The Mean and Variance of GGD with p.g.f (2) is given by*

$$Mean = \frac{q}{\theta}\delta_1 = \nu, say \tag{8}$$

and

$$Variance = \frac{4q^2}{\theta(\theta+1)}\delta_2 + \nu - \nu^2, \tag{9}$$

Proof follows from the fact that

$$Mean = E(X) = \frac{\partial H(t)}{\partial t}|_{t=1}$$

and

$$E(X(X-1)) = \frac{\partial^2 H(t)}{\partial t^2}|_{t=1}$$

.

Proposition 2.5. *For $0 < \theta < q < 1$ the mode x_0 of the GGD is the following, in which $\rho = \frac{q-\theta}{1-q}$, and [a] denote the integer part of a, for any $a \in \Re$.*

$$x_0 = \begin{cases} \rho \; and \; \rho+1 & if \quad \rho \in Z = \{0,1,2,...\} \\ [\rho+1] & if \quad \rho > 0, not \; an \; integer \end{cases} \tag{10}$$

Proof. By definition, the mode x_0 of the GGD with p.m.f g(x) is the value of x satisfying the inequalities

$$g(x) \geq g(x-1) \; and \; g(x) \geq g(x+1).$$

Now, g(x) ≥ g(x-1) implies

$$x \leq \frac{q-\theta}{1-q} + 1 = \rho + 1 \tag{11}$$

and g(x) ≥ g(x+1) implies

$$x \geq \frac{q-\theta}{1-q}. \tag{12}$$

Inequalities (11) and (12), gives the mode of the GGD as given in (10). □

Proposition 2.6. *The GGD with p.m.f g(x) as given in (1) is log concave only when* $1 \geq \theta$*.*

Proof. Proof is straightforward in the light of the inequality:

$$g^2(x+1) \geq g(x)\ g(x+2).$$

□

Proposition 2.7. *For any* $x \in \Re$*, the survival function S(x) and hazard function h(x) of the GGD are respectively*

$$S(x) = \Delta_0\ q^{x+1} \frac{(2)_x}{\theta(\theta+1)_x}\ {}_2F_1(1, 2+x; \theta+x+1; q)$$

and

$$h(x) = \frac{(1)_x\ (\theta+x)}{(2)_x\ q}\ \left[{}_2F_1(1, 2+x; \theta+x+1; q)\right]^{-1}. \tag{13}$$

Proof. The proof follows from(1), (5) and the definition of the survival function $S(x) = P(X > x)$ and hazard function $h(x) = \frac{g(x)}{S(x)}$. □

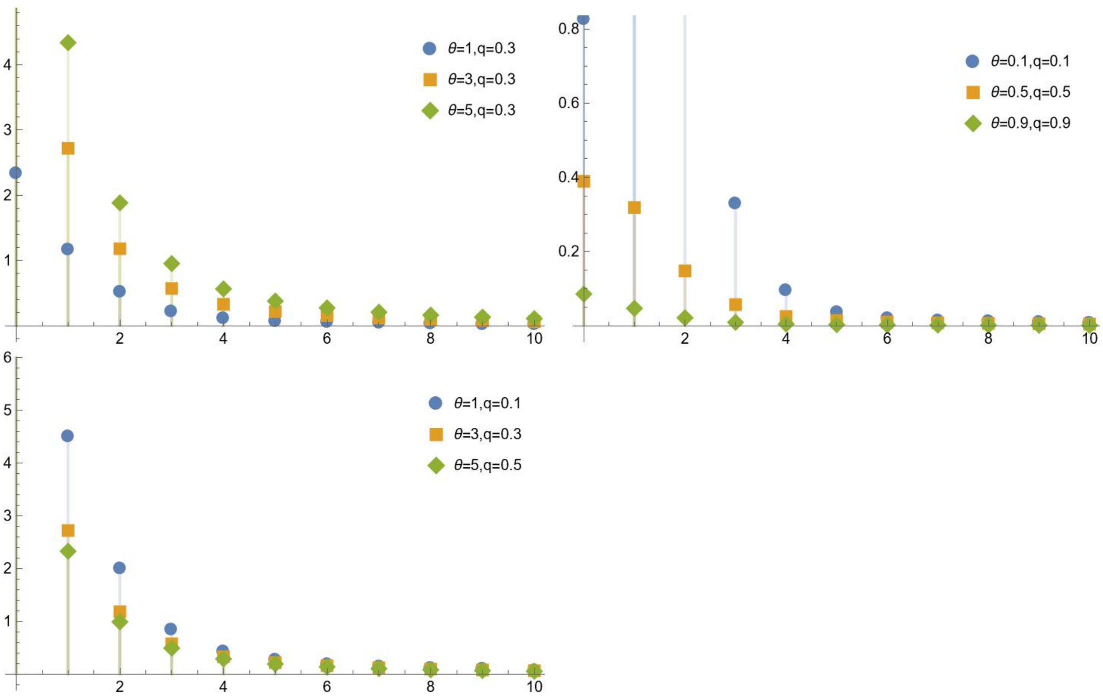

Figure 2. Illustration of the hazard of GGD for different values of θ and q.

3. Recurrence relations

Let X be a random variable having the GGD with p.g.f (2). The p.m.f g(x) of the GGD given in (1) is hereafter denoted as $g(x;\theta^*)$, where $\theta^* = (1,1;\theta)$. Define following notations for j= 0,1,2,... .

$$\theta^* + j = (1+j, 1+j; \theta + j)$$

and

$$\theta^{(j)} = \frac{1+j, 1+j}{\theta + j}.$$

Then we have the following results in the light of relations:

$$H(t) = \sum_{x=0}^{\infty} g(x;\theta^*) t^x = \Delta_0 \ {}_2F_1(1,1;\theta;qt), \tag{14}$$

and

$$\sum_{x=0}^{\infty} g(x;\theta^*+1) t^x = \Delta_1 \ {}_2F_1(2,2;\theta+1;qt), \tag{15}$$

Proposition 3.1. *A simple recurrence relation for probabilities of GGD is given by*

$$g(x+1;\theta^*) = \frac{g(x;\theta^*+1)}{(x+1)} \ \nu \tag{16}$$

Proof. On differentiating (14) with respect to t, we get

$$\sum_{x=0}^{\infty} g(x;\theta^*) \ x \ t^{x-1} = \Delta_0 \ \frac{q}{\theta} \ {}_2F_1(2,2;\theta+1;qt). \tag{17}$$

Expressions (15) and (14) together lead to the following.

$$\sum_{x=0}^{\infty} g(x+1;\theta^*) \ (x+1) \ t^x = \delta_1 \ \frac{q}{\theta} \sum_{x=0}^{\infty} g(x;\theta^*+1) \ t^x \tag{18}$$

On equating coefficient of t^x on both side of (18) we ge (16). □

Proposition 3.2. *A recurrence relation for raw moments of the GGD is given by*

$$\mu_{x+1}(\theta^*) = \nu \ \sum_{k=0}^{x} \binom{x}{k} \mu_{x-k}(\theta^*+1). \tag{19}$$

Proof. The characteristic function of GGD with p.g.f (2) has the following series representation.

$$\phi(t) = H(e^{it}) = \Delta_0\ {}_2F_1(1,1;\theta;qe^{it}) = \sum_{x=0}^{\infty} \mu_x(\theta^*)\frac{(it)^x}{x!} \tag{20}$$

From (20) we have

$$\Delta_1\ {}_2F_1(2,2;\theta+1;qe^{it}) = \sum_{x=0}^{\infty} \mu_x(\theta^*+1)\frac{(it)^x}{x!}. \tag{21}$$

Differentiate (20) with respect to t to get

$$\frac{q}{\theta}\delta_1\ e^{it}\sum_{x=0}^{\infty} \mu_x(\theta^*+1)\frac{(it)^x}{x!} = \sum_{x=0}^{\infty} \mu_x(\theta^*)\ \frac{(it)^{x-1}}{(x-1)!}. \tag{22}$$

By using (20) and (21), equation (22) become

$$\sum_{x=0}^{\infty} \mu_{x+1}(\theta^*)\frac{(it)^x}{x!} = \frac{q}{\theta}\delta_1\sum_{k=0}^{\infty}\frac{(it)^k}{k!}\sum_{x=0}^{\infty} \mu_x(\theta^*+1)\frac{(it)^x}{x!}. \tag{23}$$

Equating the coefficients of $\frac{(it)^x}{x!}$ on both sides of (23) we get (19). □

Proposition 3.3. *Recurrence relation for factorial moments of the GGD with p.g.f (2) is given by*

$$\mu_{[x+1]}(\theta^*) = \nu\ \mu_{[x]}(\theta^*+1). \tag{24}$$

Proof. The factorial moment generating function $F_X(t)$ of the GGD with p.g.f (2) has the following series representation.

$$F_X(t) = H(1+t) = \Delta_0\ {}_2F_1(1,1;\theta;q+qt) = \sum_{x=0}^{\infty} \mu_x(\theta^*)\ \frac{t^x}{x!} \tag{25}$$

From (25) we have

$$\sum_{x=0}^{\infty} \mu_x(\theta^*+1)\ \frac{t^x}{x!} = \Delta_1\ {}_2F_1(2,2;\theta+1;q+qt). \tag{26}$$

On differentiating (25) with respect to t, we have

$$\sum_{x=0}^{\infty} \mu_x(\theta^*)\ \frac{t^{x-1}}{(x-1)!} = \frac{q}{\theta}\delta_1\ \sum_{x=0}^{\infty} \mu_x(\theta^*+1)\ \frac{t^x}{x!}. \tag{27}$$

By using (25) and (26) we obtain the following from (27).

$$\sum_{x=0}^{\infty} \mu_{x+1}(\theta^*) \frac{t^x}{x!} = \frac{q}{\theta}\delta_1 \sum_{x=0}^{\infty} \mu_x(\theta^* + 1) \frac{t^x}{x!} \tag{28}$$

Equating the coefficients of $\frac{t^x}{x!}$ in (28), we get (24). □

4. Estimation

In this section, we discuss the estimation of the parameters of the GGD by the method of maximum likelihood Here we first consider the method of maximum likelihood for estimating the parameters θ and q of GGD and thereafter discussed the construction of a generalized likelihood ratio test (GLRT) procedure for testing the significance of the parameter θ . Let a(x) be the observed frequency of x events and y be the highest value of x observed. Then the likelihood function of the sample is

$$L = \prod_{x=0}^{y} [g(x)]^{a(x)},$$

which implies

$$\log L = \sum_{x=0}^{y} a(x)\ log\ g(x).$$

That is,

$$l = \log L = \sum_{x=0}^{y} a(x) \left[log\ x! + log\Gamma(\theta) + x\ loq\ q - log\Delta_0 - log\Gamma(\theta + x)\right]. \tag{29}$$

Now, on differentiating (29) with respect to the parameters θ and q, we obtain the following likelihood equations, in which $\psi(w) = \frac{\partial}{\partial w} log\Gamma(w)$. That is,

$$\frac{\partial l}{\partial\ \theta} = 0$$

implies

$$\sum_{x=0}^{y} a(x) \left[\psi(\theta) - \sum_{x=0}^{\infty} \frac{(1)_x}{(\theta_x)}\ q^x\ [\psi(\theta) - \psi(\theta + x)] - \psi(\theta + x)\right] = 0, \tag{30}$$

and

$$\frac{\partial l}{\partial\ q} = 0$$

implies

$$\sum_{x=0}^{y} a(x)\left[\frac{x}{q} - \frac{1}{\theta}\ {}_2F_1(2,2;\theta+1;q)\right] = 0, \tag{31}$$

Now, on solving the likelihood equations (30) and (31) by using some mathematical softwares like MATHEMATICA, one can obtain the maximum likelihood estimators of the parameters of the GGD.

Testing

Next we construct a test procedure for testing significance of the parameter θ of the GGD. In this regard we consider the following GLRT procedure. Here the null hypothesis is

$$H_0 : \theta = 1 \;\; against \;\; the\ alternative\ hypothesis\ H_1 : \theta \neq 1$$

Then,the test statistic is

$$-2ln\ \Lambda = 2(l_1 - l_2), \tag{32}$$

where $l_1 = lnL(\hat{\Theta}; x)$, where $\hat{\Theta}$ is the maximum likelihood estimator for $\lambda = (\theta, q)$ with no restrictions, and $l_2 = lnL(\hat{\Theta}^*; x)$, in which $\hat{\Theta}^*$ is the maximum likelihood estimator for λ under the null hypothesis H_0. The test statistic defined in (32) is asymptotically distributed as χ^2 with one degree of freedom(df). For details see Rao(1973).

5. Applications

To demonstrate the practical usefulness of the proposed generalized geometric distribution (GGD), we analyze three real medical data sets related to COVID-19 daily death counts. For each data set, the performance of the GGD is compared with two well-known competing models, namely the geometric distribution (GD) and the negative binomial distribution (NBD). The adequacy of the fitted models is evaluated using several goodness-of-fit measures, including the chi-square statistic along with its corresponding p-value, Akaike information criterion (AIC), corrected AIC (AICc), and Bayesian information criterion (BIC).

The first data set consists of the daily number of new deaths recorded over 81 days from 1 April to 20 June 2020 in El Salvador (See Almetwally and Nadarajh 2023). The second data set represents the daily new deaths observed over 81 days from 1 April to 20 May 2020 in Estonia (See Almetwally and Nadarajh 2023). The third data set corresponds to the daily new deaths reported over 81 days from 1 April to 20 May 2020 in Greece (See Almetwally and Nadarajh 2023). These data sets have been widely used in the literature for illustrating the performance of discrete lifetime and count models.

For each data set, maximum likelihood estimates of the model parameters are obtained for all competing distributions. The observed and fitted frequencies for the three models are reported in Tables 1–3. From the results, it is evident that the proposed GGD consistently produces smaller chi-square values and larger p-values when compared with the GD and NBD. This indicates a superior agreement between the observed and expected frequencies under the GGD model.

Furthermore, the information criteria values reported in the tables also support the superiority of the GGD. In all three cases, the GGD yields the minimum AIC, AICc, and BIC values among the competing models, confirming its better balance between goodness of fit and model complexity. The fitted frequency curves displayed in Figure 3 further illustrate that the GGD tracks the observed data more closely than the GD and NBD.

To test the significance of the additional parameter θ, the generalized likelihood ratio test is performed for each data set. The calculated test statistics are presented in Table 4. Since all the obtained values exceed the critical chi-square value at the 5% level of significance with one degree of freedom, the null hypothesis $\theta = 1$ is rejected in all cases. This clearly establishes the statistical significance of the additional parameter and justifies the use of the generalized model over the classical geometric distribution.

Table 1. Distribution of the daily new deaths of 81 days from 1 April to 20 June 2020 in El Salvador

X	Observed Frequency	NBD	GD	GGD
0	34	33	40	34
1	25	26	21	25
2	11	13	10	12
3	6	5	4	6
4	4	2	2	3
5	1	2	3	1
Total	81	81	81	81
df		1	4	1
Estimates		$n = 2.33$	$p = 0.48$	$\theta = 0.51$
		$p = 0.68$		$q = 0.37$
χ^2-value		3.076	6.095	0.416
P-value		0.079	0.192	0.51
AIC		233.32	233.36	232.82
BIC		233.13	233.27	232.63
AICc		233.47	233.68	232.97

Table 2. Distribution of the daily new deaths of 81 days from 1 April to 20 May 2020 in Estonia

X	Observed Frequency	NBD	GD	GGD
0	102	103	106	102
1	39	37	33	38
2	7	10	10	8
3	4	2	3	4
4	1	3	1	1
Total	153	153	153	153
df		1	1	1
Estimates		$n = 1.71$	$p = 0.68$	$\theta = 0.61$
		$p = 0.79$		$q = 0.22$
χ^2-value		4.3511	2.4751	0.1513
P-value		0.0360	0.1156	0.6972
AIC		280.24	279.16	278.18
BIC		280.60	279.34	278.54
AICc		280.32	279.18	278.26

Table 3. Distribution of the daily new deaths of 81 days from 1 April to 20 May 2020 in Greece

X	Observed Frequency	NBD	GD	GGD
0	39	40	40	38
1	26	27	25	27
2	17	18	16	17
3	9	11	11	10
4	6	5	7	7
5	7	4	4	5
6	6	2	3	5
7	0	2	2	0.5
8	0	1	2	0.5
9	1	1	1	1
Total	153	153	153	153
df		4	5	4
Estimates		$n = 1.22$	$p = 0.36$	$\theta = 0.83$
		$p = 0.41$		$q = 0.59$
χ^2-value		13.9312	9.8839	2.3060
P-value		0.0075	0.0785	0.6796
AIC		402.64	401.21	400.68
BIC		402.73	401.25	400.77
AICc		402.75	401.27	400.79

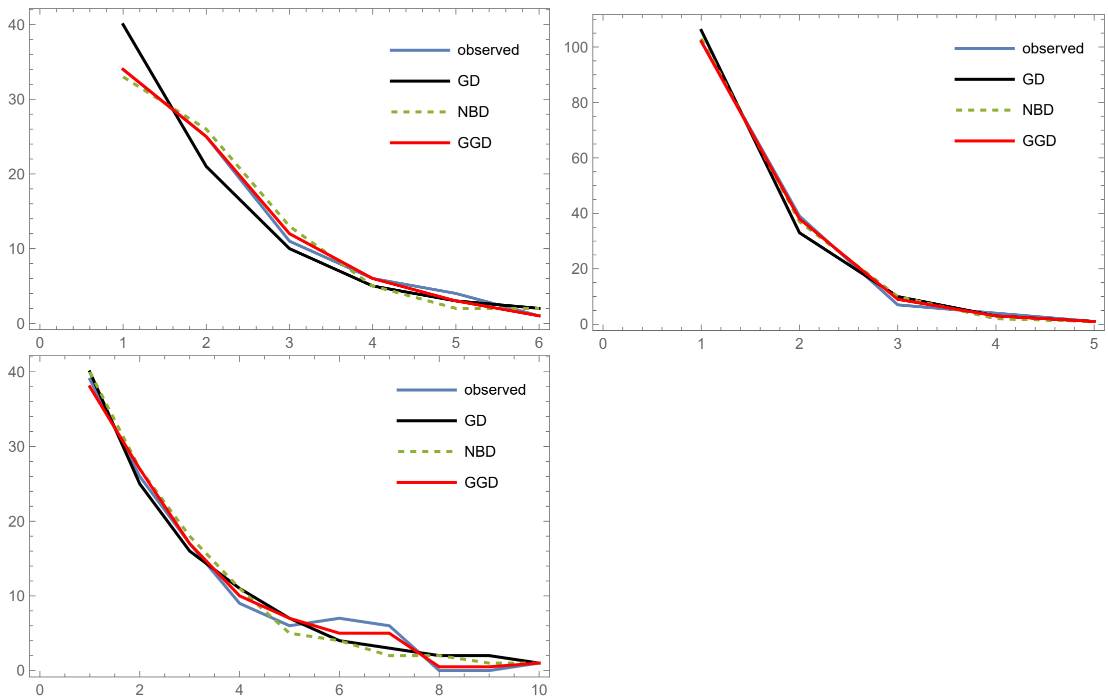

Figure 3. Frequency curves corresponding to various models based on data set 1, data set 2 and data set 3.

Table 4. Calculated value of the test statistic in case of the GLRT

Data Set	$\ln L(\hat{\Theta}^{*}; x)$	$\ln L(\hat{\Theta}; x)$	Test Statistic
1	-116.68	-114.41	4.54
2	-139.58	-137.09	4.98
3	-199.60	-197.34	4.52

6. Simulation

In this section, our main aim is to compare the theoretical performance of the estimators of different parameters of the GGD obtained by the method of maximum likelihood. On the basis of a brief simulation study. Here we have simulated data sets for two sets of parameters as shown in Table 5, and computed the absolute bias and standard errors in case of each simulated sample. The simulation results are summarised in Table 5 corresponding to the sample size 100, 300 and 500 for the following two parameter sets.

1. $\theta = 2$, q $= 0.5$ (Over-dispersed)
2. $\theta = 0.05$, q $= 0.2$ (Under-dispersed)

From Table 5, it can be seen that as the sample size increases, both absolute bias and standard errors corresponding to the parameter sets are in decreasing order in all the cases.

Table 5. Absolute bias and standard errors in parentheses of the estimators $\hat{\theta}$ and $\hat{q}$ of the GGD for simulated data sets.

Parameter Set	Sample Size	$\hat{\theta}$	$\hat{q}$
(i)	$n = 100$	-0.31 (0.75)	-0.12 (0.12)
	$n = 300$	-0.53 (0.28)	-0.24 (0.06)
	$n = 500$	-0.83 (0.07)	-0.33 (0.01)
(ii)	$n = 100$	0.01 (0.029)	0.01 (0.02)
	$n = 300$	-0.003 (0.005)	-0.04 (0.0003)
	$n = 500$	-0.02 (0.001)	-0.14 (0.0002)

7. Conclusion

In this study, a two-parameter generalization of the geometric distribution has been proposed and several of its important statistical properties have been derived in explicit analytical form. Expressions for the probability generating function, cumulative distribution function, moments, mode, recurrence relations, survival function, and hazard rate function were obtained. Parameter estimation was carried out using the method of maximum likelihood, and a generalized likelihood ratio test was developed to assess the significance of the additional shape parameter.

The practical relevance of the proposed model was illustrated using three real COVID-19 related medical data sets. A comparative analysis with the geometric and negative binomial distributions clearly showed that the generalized geometric distribution provides a significantly better fit in all considered cases. The superiority of the model was strongly supported by goodness-of-fit tests as well as model selection criteria such as AIC, AICc, and BIC.

A brief simulation study further confirmed the satisfactory performance of the maximum likelihood estimators in terms of bias and standard error, especially as the sample size increases. These findings establish that the proposed generalized geometric distribution is a flexible and reliable model for analyzing medical and epidemiological count data exhibiting different dispersion patterns.

Overall, the results of this study suggest that the proposed model can serve as a useful alternative to existing discrete distributions in practical data analysis. The distribution may also find applications in other areas such as reliability engineering, actuarial science, and insurance studies where geometric-type count data frequently arise.

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